

# Creating a Unified Graphical Wind Turbulence Model from Multiple Specifications

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## 1.0 Abstract

*This paper documents the research for and development of the unified wind turbulence model block for a Commercial Off-the-Shelf (COTS) software package that allows for a graphical hierarchical block diagram representation of entire vehicles including environment and control models. Recognized standards were used as a basis for the COTS wind turbulence block's operational requirements. Standards used were MIL-F-8785C and MIL-HDBK-1797, which provide guidelines for modeling wind turbulence. Additionally multiple papers on wind turbulence were referenced.*

*This paper is not only a collection of information from different versions of the standards and multiple papers, but also an interpretation and a integration of this information. This research involves the continuous Von Kármán turbulence, the continuous Dryden turbulence, and the discrete Dryden turbulence. There are many topics to be covered in documenting this research. Some topics of interest are the multiple interpretations of the angular rate filters, the variation in scale lengths in the different versions of the recognized standards, and their effects on the Von Kármán and Dryden filters. Other topics to be discussed in this paper are the application of the low altitude model, the application of the medium/high altitude model, and, particularly, a method for transitioning between the two models. A concluding topic discusses how these differences in the standards can be integrated into a single unified graphical model.*

## 2.0 Symbols

$\omega$	Temporal frequency
$b$	Aircraft wingspan
$h$	Altitude
$u_{20}$	Wind speed at 20 feet
$u_g$	Disturbance velocity along the x-axis
$v_g$	Disturbance velocity along the y-axis
$w_g$	Disturbance velocity along the z-axis
$p_g$	Disturbance angular velocity about the x-axis
$q_g$	Disturbance angular velocity about the y-axis
$r_g$	Disturbance angular velocity about the z-axis
$L_u$	Scale length of $u_g$
$L_v$	Scale length of $v_g$
$L_w$	Scale length of $w_g$
$\sigma_u$	Root-mean-square intensity of $u_g$
$\sigma_v$	Root-mean-square intensity of $v_g$
$\sigma_w$	Root-mean-square intensity of $w_g$
$\Phi_u$	Spectrum for $u_g$
$\Phi_v$	Spectrum for $v_g$
$\Phi_w$	Spectrum for $w_g$
$\Phi_p$	Spectrum for $p_g$
$\Phi_q$	Spectrum for $q_g$
$\Phi_r$	Spectrum for $r_g$

## 3.0 Interpretation of the Standards

The military guidelines for modeling wind turbulence are complicated partially because their implementation is not always obvious. Therefore, to use the guidelines properly, much analysis and interpretation of the guidelines is required. For those who are not directly interested in the study of wind turbulence, this paper provides the unique service of being a compilation of military standard wind turbulence information and implementation. Much of the information needed for a basic understanding and implementation of wind turbulence models defined by military guidelines is contained in the following sections.

### 3.0.1 Definitions of Turbulence Spectra

According to the military references[1, 2], turbulence is a stochastic process defined by velocity spectra. The turbulence field is assumed to be visualized as frozen in time and space (i.e.: time variations are statistically equivalent to distance variations in traversing the turbulence field). This assumption implies the turbulence-induced responses of the aircraft result only from the motion of the aircraft relative to the turbulent field.

In the military references[1, 2], either the Dryden spectral representation or the Von Kármán spectral representation can be used to generate turbulence by filtering band-limited white noise with an appropriate forming filter derived from the spectral representation.

The following two tables display the longitudinal spectra functions for Dryden and Von Kármán turbulence from both military references.

	MIL-F-8785C	MIL-HDBK-1797
$\Phi_u(\omega)$	$\frac{2\sigma_u^2 L_u}{\pi V} \cdot \frac{1}{1 + (L_u \frac{\omega}{V})^2}$	$\frac{2\sigma_u^2 L_u}{\pi V} \cdot \frac{1}{1 + (L_u \frac{\omega}{V})^2}$
$\Phi_p(\omega)$	$\frac{\sigma_w^2}{VL_w} \cdot \frac{0.8 \left(\frac{\pi L_w}{4b}\right)^{\frac{1}{3}}}{1 + \left(\frac{4b\omega}{\pi V}\right)^2}$	$\frac{\sigma_w^2}{VL_w} \cdot \frac{0.8 \left(\frac{\pi L_w}{4b}\right)^{\frac{1}{3}}}{1 + \left(\frac{4b\omega}{\pi V}\right)^2}$

Table 1: Dryden Longitudinal Spectra

	MIL-F-8785C	MIL-HDBK-1797
$\Phi_u(\omega)$	$\frac{2\sigma_u^2 L_u}{\pi V} \cdot \frac{1}{\left[1 + (1.339 L_u \frac{\omega}{V})^2\right]^{5/6}}$	$\frac{2\sigma_u^2 L_u}{\pi V} \cdot \frac{1}{\left[1 + (1.339 L_u \frac{\omega}{V})^2\right]^{5/6}}$
$\Phi_p(\omega)$	$\frac{\sigma_w^2}{VL_w} \cdot \frac{0.8 \left(\frac{\pi L_w}{4b}\right)^{\frac{1}{3}}}{1 + \left(\frac{4b\omega}{\pi V}\right)^2}$	$\frac{\sigma_w^2}{2VL_w} \cdot \frac{0.8 \left(\frac{2\pi L_w}{4b}\right)^{\frac{1}{3}}}{1 + \left(\frac{4b\omega}{\pi V}\right)^2}$

Table 2: Von Kármán Longitudinal Spectra

The following two tables display the lateral spectra functions for Dryden and Von Kármán turbulence from both military references.

	MIL-F-8785C	MIL-HDBK-1797
$\Phi_v(\omega)$	$\frac{\sigma_v^2 L_v}{\pi V} \cdot \frac{1 + 3(L_v \frac{\omega}{V})^2}{\left[1 + (L_v \frac{\omega}{V})^2\right]^2}$	$\frac{2\sigma_v^2 L_v}{\pi V} \cdot \frac{1 + 12(L_v \frac{\omega}{V})^2}{\left[1 + 4(L_v \frac{\omega}{V})^2\right]^2}$
$\Phi_r(\omega)$	$\frac{\mp \left(\frac{\omega}{V}\right)^2}{1 + \left(\frac{3b\omega}{\pi V}\right)^2} \cdot \Phi_v(\omega)$	$\frac{\mp \left(\frac{\omega}{V}\right)^2}{1 + \left(\frac{3b\omega}{\pi V}\right)^2} \cdot \Phi_v(\omega)$

Table 3: Dryden Lateral Spectra

	MIL-F-8785C	MIL-HDBK-1797
$\Phi_v(\omega)$	$\frac{\sigma_v^2 L_v}{\pi V} \cdot \frac{1 + \frac{8}{3} (1.339 L_v \frac{\omega}{V})^2}{\left[1 + (1.339 L_v \frac{\omega}{V})^2\right]^{11/6}}$	$\frac{2\sigma_v^2 L_v}{\pi V} \cdot \frac{1 + \frac{8}{3} (2.678 L_v \frac{\omega}{V})^2}{\left[1 + (2.678 L_v \frac{\omega}{V})^2\right]^{11/6}}$
$\Phi_r(\omega)$	$\frac{\mp \left(\frac{\omega}{V}\right)^2}{1 + \left(\frac{3b\omega}{\pi V}\right)^2} \cdot \Phi_v(\omega)$	$\frac{\mp \left(\frac{\omega}{V}\right)^2}{1 + \left(\frac{3b\omega}{\pi V}\right)^2} \cdot \Phi_v(\omega)$

Table 4: Von Kármán Lateral Spectra

The following two tables display the vertical spectra functions for Dryden and Von Kármán turbulence from both military references.

	MIL-F-8785C	MIL-HDBK-1797
$\Phi_w(\omega)$	$\frac{\sigma_w^2 L_w}{\pi V} \cdot \frac{1 + 3(L_w \frac{\omega}{V})^2}{\left[1 + (L_w \frac{\omega}{V})^2\right]^2}$	$\frac{2\sigma_w^2 L_w}{\pi V} \cdot \frac{1 + 12(L_w \frac{\omega}{V})^2}{\left[1 + 4(L_w \frac{\omega}{V})^2\right]^2}$
$\Phi_q(\omega)$	$\frac{\pm \left(\frac{\omega}{V}\right)^2}{1 + \left(\frac{4b\omega}{\pi V}\right)^2} \cdot \Phi_w(\omega)$	$\frac{\pm \left(\frac{\omega}{V}\right)^2}{1 + \left(\frac{4b\omega}{\pi V}\right)^2} \cdot \Phi_w(\omega)$

Table 5: Dryden Vertical Spectra

	MIL-F-8785C	MIL-HDBK-1797
$\Phi_w(\omega)$	$\frac{\sigma_w^2 L_w}{\pi V} \cdot \frac{1 + \frac{8}{3} (1.339 L_w \frac{\omega}{V})^2}{\left[1 + (1.339 L_w \frac{\omega}{V})^2\right]^{11/6}}$	$\frac{2\sigma_w^2 L_w}{\pi V} \cdot \frac{1 + \frac{8}{3} (2.678 L_w \frac{\omega}{V})^2}{\left[1 + (2.678 L_w \frac{\omega}{V})^2\right]^{11/6}}$
$\Phi_q(\omega)$	$\frac{\pm \left(\frac{\omega}{V}\right)^2}{1 + \left(\frac{4b\omega}{\pi V}\right)^2} \cdot \Phi_w(\omega)$	$\frac{\pm \left(\frac{\omega}{V}\right)^2}{1 + \left(\frac{4b\omega}{\pi V}\right)^2} \cdot \Phi_w(\omega)$

Table 6: Von Kármán Vertical Spectra

Examining the previous tables, minor differences can be seen in the spectra from MIL-F-8785C[2] and MIL-HDBK-1797[1]. These differences are due to changes to the lateral and vertical scale lengths in the two references. An additional notable detail is the sign of the lateral and vertical angular rate spectra. This is due to multiple approximations of turbulence angular rates.

### 3.1 Multiple Approximations of Angular Rates

There are multiple definitions of the angular rate filters due to multiple approximations of turbulence angular rates. The spectral density definitions are dependent on these approximations of turbulence angular rates that are defined in the references as three variations. These turbulence angular rates variations are displayed in the following table.

MIL-F-8785C & MIL-HDBK-1797 (Notice 1)	MIL-HDBK-1797	Reference [6]
$p_g = \frac{\partial w_g}{\partial y}$	$p_g = \frac{\partial w_g}{\partial y}$	$p_g = -\frac{\partial w_g}{\partial y}$
$q_g = \frac{\partial w_g}{\partial x}$	$q_g = \frac{\partial w_g}{\partial x}$	$q_g = -\frac{\partial w_g}{\partial x}$
$r_g = -\frac{\partial v_g}{\partial x}$	$r_g = \frac{\partial v_g}{\partial x}$	$r_g = \frac{\partial v_g}{\partial x}$

Table 7: Angular Rate Approximations

The variations in definitions affect only the vertical ( $q_g$ ) and lateral ( $r_g$ ) turbulence angular rates, since the longitudinal turbulence angular rate spectrum,  $\Phi_p(\omega)$ , is a rational function. This rational function is derived from curve-fitting a complex algebraic function, not the vertical turbulence velocity spectrum,  $\Phi_w(\omega)$ , multiplied by a scale factor. Because the turbulence angular rate spectra contribute less to the aircraft gust response than the turbulence velocity spectra, it may explain the variations in the angular rate definitions.

The turbulence angular rate variations lead to the following table of sign combinations for vertical and lateral turbulence angular rate spectra. These sign combinations were

	MIL-F-8785C & MIL-HDBK-1797 (Notice 1)	MIL-HDBK-1797	Reference [6]
<b>Vertical</b>	$\Phi_q(\omega)$	$\Phi_q(\omega)$	$-\Phi_q(\omega)$
<b>Lateral</b>	$-\Phi_r(\omega)$	$\Phi_r(\omega)$	$\Phi_r(\omega)$

Table 8: Signed Vertical and Lateral Spectra

previously represented in the lateral and vertical spectra tables (tables 4 through 6). The selection of sign combinations for the lateral and vertical turbulence angular rate spectra directly affects the sign of the resulting angular rate filters. These resulting filters can be seen in the next section.

### 3.2 Resulting Velocity and Angular Rate Filters

To generate a turbulence signal with the correct characteristics, a unit variance, band-limited white noise signal is passed through or used in the appropriate filters. These filters are ultimately derived from the turbulence spectra. Three forms of the filters are covered in the military references[1,2]

- Continuous Von Kármán,
- Continuous Dryden, and
- Discrete Dryden.

The details of each type of filter are discussed in the following paragraphs.

#### 3.2.1 Continuous Von Kármán Filters

To use the continuous Von Kármán filter properly, the unit variance, band-limited white noise signal is passed through the Von Kármán forming filters. Deriving forming filters requires that the spectral square roots be obtainable from the spectrum equations. Since the Von Kármán spectra are not spectrally factorable, the Von Kármán spectra must be curve-fitted to a satisfactory degree of approximation with a factorable spectral form for which transfer functions may be obtained[3]. The Von Kármán forming filters are then derived from the spectral square roots of the approximated spectrum equations.

Defined in references [1] and [5], the forming filters, approximations of the Von Kármán velocity spectra, are valid in a range of normalized frequencies of less than 50 radians.

<b>MIL-F-8785C &amp; MIL-HDBK-1797</b>	
$H_u(s)$	$\frac{\sigma_u \sqrt{\frac{2L_u}{\pi V}} \left(1 + 0.25 \frac{L_u}{V} s\right)}{1 + 1.357 \frac{L_u}{V} s + 0.1987 \left(\frac{L_u}{V}\right)^2 s^2}$
$H_p(s)$	$\sigma_w \sqrt{\frac{0.8}{V}} \frac{\left(\frac{\pi}{4b}\right)^{1/6}}{L_w^{1/3} \left(1 + \left(\frac{4b}{\pi V}\right) s\right)}$
$H_v(s)$	$\frac{\sigma_v \sqrt{\frac{L_v}{\pi V}} \left(1 + 2.7478 \frac{L_v}{V} s + 0.3398 \left(\frac{L_v}{V}\right)^2 s^2\right)}{1 + 2.9958 \frac{L_v}{V} s + 1.9754 \left(\frac{L_v}{V}\right)^2 s^2 + 0.1539 \left(\frac{L_v}{V}\right)^3 s^3}$
$H_r(s)$	$\frac{\mp \frac{s}{V}}{\left(1 + \left(\frac{3b}{\pi V}\right) s\right)} \cdot H_v(s)$
$H_w(s)$	$\frac{\sigma_w \sqrt{\frac{L_w}{\pi V}} \left(1 + 2.7478 \frac{L_w}{V} s + 0.3398 \left(\frac{L_w}{V}\right)^2 s^2\right)}{1 + 2.9958 \frac{L_w}{V} s + 1.9754 \left(\frac{L_w}{V}\right)^2 s^2 + 0.1539 \left(\frac{L_w}{V}\right)^3 s^3}$
$H_q(s)$	$\frac{\pm \frac{s}{V}}{\left(1 + \left(\frac{4b}{\pi V}\right) s\right)} \cdot H_w(s)$

Table 9: Continuous Von Kármán Turbulence Transfer Functions

An interesting side note is that since the Von Kármán turbulence spectra must be curve-fitted, it is possible to create alternative or higher-order forming filters. These alternatives are not covered in the military references[1,2].

### 3.2.2 Continuous Dryden Filters

To use the continuous Dryden filter properly, the unit variance, band-limited white noise signal is passed through the Dryden forming filters. Deriving forming filters requires that the spectral square roots be obtainable from the spectrum equations. Since the Dryden spectra are spectrally factorable,

the Dryden forming filters are then derived from the spectral square roots of the Dryden spectrum equations.

	<b>MIL-F-8785C</b>	<b>MIL-HDBK-1797</b>
$H_u(s)$	$\sigma_u \sqrt{\frac{2L_u}{\pi V}} \frac{1}{1 + \frac{L_u}{V} s}$	$\sigma_u \sqrt{\frac{2L_u}{\pi V}} \frac{1}{1 + \frac{L_u}{V} s}$
$H_p(s)$	$\sigma_w \sqrt{\frac{0.8}{V}} \frac{\left(\frac{\pi}{4b}\right)^{1/6}}{L_w^{1/3} \left(1 + \left(\frac{4b}{\pi V}\right) s\right)}$	$\sigma_w \sqrt{\frac{0.8}{V}} \frac{\left(\frac{\pi}{4b}\right)^{1/6}}{(2L_w)^{1/3} \left(1 + \left(\frac{4b}{\pi V}\right) s\right)}$
$H_v(s)$	$\sigma_v \sqrt{\frac{L_v}{\pi V}} \frac{1 + \frac{\sqrt{3}L_v}{V} s}{\left(1 + \frac{L_v}{V} s\right)^2}$	$\sigma_v \sqrt{\frac{2L_v}{\pi V}} \frac{1 + \frac{2\sqrt{3}L_v}{V} s}{\left(1 + \frac{2L_v}{V} s\right)^2}$
$H_r(s)$	$\frac{\mp \frac{s}{V}}{\left(1 + \left(\frac{3b}{\pi V}\right) s\right)} \cdot H_v(s)$	$\frac{\mp \frac{s}{V}}{\left(1 + \left(\frac{3b}{\pi V}\right) s\right)} \cdot H_v(s)$
$H_w(s)$	$\sigma_w \sqrt{\frac{L_w}{\pi V}} \frac{1 + \frac{\sqrt{3}L_w}{V} s}{\left(1 + \frac{L_w}{V} s\right)^2}$	$\sigma_w \sqrt{\frac{2L_w}{\pi V}} \frac{1 + \frac{2\sqrt{3}L_w}{V} s}{\left(1 + \frac{2L_w}{V} s\right)^2}$
$H_q(s)$	$\frac{\pm \frac{s}{V}}{\left(1 + \left(\frac{4b}{\pi V}\right) s\right)} \cdot H_w(s)$	$\frac{\pm \frac{s}{V}}{\left(1 + \left(\frac{4b}{\pi V}\right) s\right)} \cdot H_w(s)$

Table 10: Continuous Dryden Turbulence Transfer Functions

### 3.2.3 Discrete Dryden Filters

To use the discrete Dryden filter properly, the unit variance, band-limited white noise signal is used in the digital filter finite difference equations. To derive the discrete Dryden finite difference equations, the euler integration method is applied to the continuous Dryden forming filters.

The following table displays the digital filter finite difference equations.

	MIL-F-8785C	MIL-HDBK-1797
$H_u(z)$	$\left(1 - \frac{V}{L_u}T\right)u_g + \sqrt{2\frac{V}{L_u}T\frac{\sigma_u}{\sigma_\eta}}\eta_1$	$\left(1 - \frac{V}{L_u}T\right)u_g + \sqrt{2\frac{V}{L_u}T\frac{\sigma_u}{\sigma_\eta}}\eta_1$
$H_p(z)$	$\left(1 - \frac{2.6}{\sqrt{L_w b}}T\right)p_g + \frac{0.95}{\sqrt{2\frac{2.6}{\sqrt{L_w b}}T\sqrt{2L_w b^2}}}\frac{\sigma_w}{\sigma_\eta}\eta_4$	$\left(1 - \frac{2.6}{\sqrt{2L_w b}}T\right)p_g + \frac{1.9}{\sqrt{2\frac{2.6}{\sqrt{L_w b}}T\sqrt{2L_w b}}}\frac{\sigma_w}{\sigma_\eta}\eta_4$
$H_v(z)$	$\left(1 - \frac{V}{L_u}T\right)v_g + \sqrt{2\frac{V}{L_u}T\frac{\sigma_v}{\sigma_\eta}}\eta_2$	$\left(1 - \frac{V}{L_u}T\right)v_g + \sqrt{2\frac{V}{L_u}T\frac{\sigma_v}{\sigma_\eta}}\eta_2$
$H_r(z)$	$\left(1 - \frac{\pi V}{3b}T\right)r_g \mp \frac{\pi}{3b}(v_g - v_{g_{pass}})$	$\left(1 - \frac{\pi V}{3b}T\right)r_g \mp \frac{\pi}{3b}(v_g - v_{g_{pass}})$
$H_w(z)$	$\left(1 - \frac{V}{L_u}T\right)w_g + \sqrt{2\frac{V}{L_u}T\frac{\sigma_w}{\sigma_\eta}}\eta_3$	$\left(1 - \frac{V}{L_u}T\right)w_g + \sqrt{2\frac{V}{L_u}T\frac{\sigma_w}{\sigma_\eta}}\eta_3$
$H_q(z)$	$\left(1 - \frac{\pi V}{4b}T\right)q_g \pm \frac{\pi}{4b}(w_g - w_{g_{pass}})$	$\left(1 - \frac{\pi V}{4b}T\right)q_g \pm \frac{\pi}{4b}(w_g - w_{g_{pass}})$

Table 11: Discrete Dryden Turbulence Difference Equations

### 3.3 Application at Altitude

The method of applying the wind turbulence models is defined in the military references[1, 2]. The turbulence model is divided into two distinct regions, low altitude and medium/high altitude, affecting the turbulence scale lengths and intensities used to generate wind velocities and angular rates. Additionally, the turbulence axes orientation also differs in these regions.

#### 3.3.1 Low Altitude Model

The low altitude model is defined as altitude less than or equal to 1000 feet. Three values are defined in this region;

- turbulence scale length,
- turbulence intensity, and
- turbulence axes orientation.

According to the military references[1, 2], the turbulence scale lengths at low altitudes are represented in the following table.

MIL-F-8785C	MIL-HDBK-1797
$L_w = h$	$2L_w = h$
$L_u = L_v = \frac{h}{(0.177 + 0.000823h)^{1.2}}$	$L_u = 2L_v = \frac{h}{(0.177 + 0.000823h)^{1.2}}$

Table 12: Low Altitude Turbulence Scale Lengths

According to the military references[1, 2], the turbulence intensities are given in the following equations.

$$\sigma_w = 0.1u_{20} \quad (1)$$

$$\frac{\sigma_u}{\sigma_w} = \frac{\sigma_v}{\sigma_w} = \frac{1}{(0.177 + 0.000823h)^{0.4}} \quad (2)$$

Typical values for wind speed at 20 feet are defined in the military references[1, 2]. These typical values are defined in the following table.

Turbulence Level	$u_{20}$
Light	15 knots
Moderate	30 knots
Severe	45 knots

Table 13: Typical Wind Speeds at 20 feet

In the low altitude region, the turbulence axes orientation is defined as

- Longitudinal turbulence velocity,  $u_g$ , along the horizontal relative mean wind vector, and
- Vertical turbulence velocity,  $w_g$ , aligned with vertical.

In this region, the output of the COTS block is transformed into body coordinates.

#### 3.3.2 Medium/High Altitude Model

The medium/high altitude is defined as altitude greater than or equal to 2000 feet. Three values are defined in this region;

- turbulence scale length,
- turbulence intensity, and
- turbulence axes orientation.

According to the military references[1, 2], for medium to high altitudes the turbulence scale lengths and intensities are based on the assumption that the turbulence is isotropic. In the

military references[1, 2], the scale lengths are represented by the following equations.

MIL-F-8785C	MIL-HDBK-1797
$L_u = L_v = L_w = 1750ft$	$L_u = 2L_v = 2L_w = 1750ft$

Table 14: Dryden Turbulence Scale Lengths

MIL-F-8785C	MIL-HDBK-1797
$L_u = L_v = L_w = 2500ft$	$L_u = 2L_v = 2L_w = 2500ft$

Table 15: Von Kármán Turbulence Scale Lengths

In the military references[1, 2], the turbulence intensities are defined in a diagram that provides the turbulence intensity as a function of altitude and the probability of the turbulence intensity being exceeded (Figure 1). These turbulence intensities are implemented in a lookup table within the COTS software.

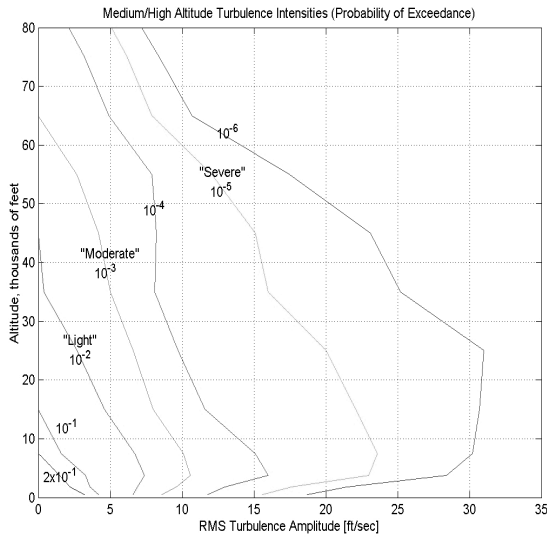


Figure 1: Medium/High Turbulence Intensities

The relationship of the turbulence intensities is represented in the following equation.

$$\sigma_u = \sigma_v = \sigma_w \quad (3)$$

In the medium/high altitude region, alignment of the turbulence axes with either the body or stability coordinates is deemed equally acceptable in references [3,7]. For the COTS block, the turbulence axes orientation is chosen to be aligned with the body coordinates.

### 3.3.3 Method for Transitioning Between Regions

For the COTS block, the altitudes in the transition region are defined as between low and medium/high altitudes, i.e. an altitude above 1000 feet and below 2000 feet. This region is undefined within the military references[1, 2]. In order provide a turbulence model that is continuous in altitude, a

method for transitioning between the two regions was developed for the COTS block.

The method developed for transitioning between low and medium/high altitudes is reasonably straightforward. At altitudes between 1000 feet and 2000 feet, the turbulence velocities and turbulence angular rates are determined by linearly interpolating between the value from the low altitude model at 1000 feet transformed from mean horizontal wind coordinates to body coordinates and the value from the high altitude model at 2000 feet in body coordinates.

### 3.4 Effects of Scale Length

The general effects of an alternate scale length value are a change in the power spectral density asymptote and gust load of the turbulence. It is acceptable to use scale lengths that differ from those specified by the military references[1,2], but the resulting turbulence does not conform to the turbulence of those references.

The effects on the turbulence filters due to changes in the scale lengths between military references[1,2] are discussed below. For the continuous Dryden filters, evaluating the turbulence scale lengths within the transfer functions results in the same final transfer functions. This is due to the factor of two differences in turbulence scale lengths and turbulence transfer functions offsetting each other. For the continuous Von Kármán filters, the references[1,5] refer to the same transfer functions for both military specifications. The turbulence scale lengths changes between military references have not impacted the form of the continuous Von Kármán turbulence transfer functions. For the discrete Dryden filters, the scale lengths do not affect the difference equations. The difference equations are defined in terms of  $L_u$ , which does not change between the military references.

### 4.0 Development of Unified Block in COTS

Using the collected information from the military references[1,2], a unified wind turbulence model block for a COTS software package, Simulink®, was developed. A general overview of the block, such as the block's user interface is given. The block's structure and method of implementing all three wind models is shown. The graphical implementation of the interpolation method for transitioning between the low and medium/high altitude models is also shown.

#### 4.1 Availability of COTS Block

The unified COTS wind turbulence model block will be available within a future release of the Aerospace Blockset

from The MathWorks, Inc. An overview of library contents of that version of Aerospace Blockset is shown in Figure 2.

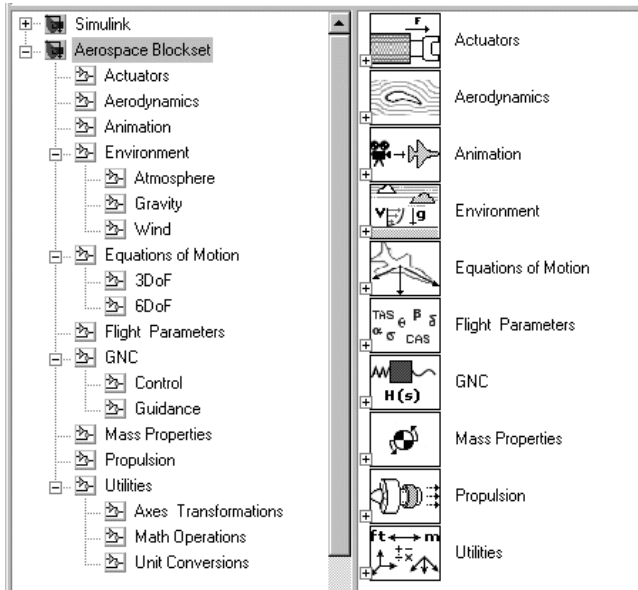


Figure 2: Aerospace Blockset Library Overview

To locate the unified wind turbulence model block, open the environment library, which exposes the atmosphere, gravity and wind libraries. By opening the wind library, the block is exposed. Using the Simulink library feature of linked blocks, all variations of the unified block are shown in Figure 3. The unified block does not include the Discrete Wind Gust Model block, the Wind Shear Model block, or the Horizontal Wind Model block, which are also shown in Figure 3.

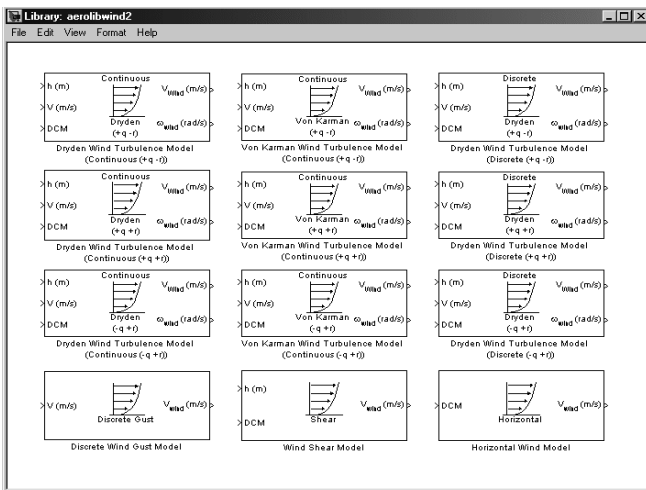


Figure 3: Wind Library

### 4.2 Block Inputs and Outputs

The various wind turbulence models have the identical inputs and outputs, which made creating a single unified block easier. The inputs to the block are

- altitude,
- aircraft speed, and

- direction cosine matrix.

The outputs of the block are

- a three-element signal containing the turbulence velocities, and
- a three-element signal containing the turbulence angular rates.

Since no adjustments are needed due to the inputs or outputs, inputs and outputs have no effect on the structure of the block.

### 4.3 Mask Parameters

Select parameters are needed to simulate the wind turbulence model. Since these parameters are the same for all models, the block dialog does not change. The parameters are set by the block dialog shown in Figure 4.

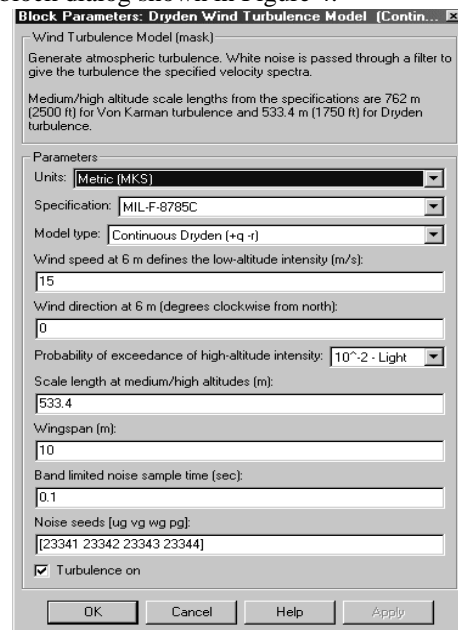


Figure 4: Wind Turbulence Dialog

Below are detailed descriptions of the parameters in the dialog.

### Units

Define the units of wind speed due to the turbulence.

	Wind Velocity	Altitude	Airspeed
Metric (MKS)	Meters/second	Meters	Meters/second
English (Velocity in ft/s)	Feet/second	Feet	Feet/second
English (Velocity in kts)	Knots	Feet	Knots

### Specification

Define which military reference to use. This affects the application of turbulence scale lengths in the lateral and vertical directions.

## Model Type

Select the window turbulence model to use.

Model	Description
Continuous Von Kármán (+q -r)	Use continuous representation of Von Kármán velocity spectra with positive vertical and negative lateral angular rates spectra.
Continuous Von Kármán (+q +r)	Use continuous representation of Von Kármán velocity spectra with positive vertical and lateral angular rates spectra.
Continuous Von Kármán (-q +r)	Use continuous representation of Von Kármán velocity spectra with negative vertical and positive lateral angular rates spectra.
Continuous Dryden (+q -r)	Use continuous representation of Dryden velocity spectra with positive vertical and negative lateral angular rates spectra.
Continuous Dryden (+q +r)	Use continuous representation of Dryden velocity spectra with positive vertical and lateral angular rates spectra.
Continuous Dryden (-q +r)	Use continuous representation of Dryden velocity spectra with negative vertical and positive lateral angular rates spectra.
Discrete Dryden (+q -r)	Use discrete representation of Dryden velocity spectra with positive vertical and negative lateral angular rates spectra.
Discrete Dryden (+q +r)	Use discrete representation of Dryden velocity spectra with positive vertical and lateral angular rates spectra.
Discrete Dryden (-q +r)	Use discrete representation of Dryden velocity spectra with negative vertical and positive lateral angular rates spectra.

## Wind speed at 6 m defines the low altitude intensity

The measured wind speed at a height of 20 feet (6 meters) provides the intensity for the low-altitude turbulence model.

## Wind direction at 6 m (degrees clockwise from north)

The measured wind direction at a height of 20 feet (6 meters) is an angle to aid in transforming the low-altitude turbulence model into a body coordinates.

## Probability of exceedance of high altitude intensity

Above 2000 feet, the turbulence intensity is determined from a lookup table that gives the turbulence intensity as a function of altitude and the probability of the turbulence intensity's being exceeded.

## Scale length at medium/high altitudes

The turbulence scale length above 2000 feet is assumed constant. From the military references, a value of 1750 feet is recommended for the longitudinal turbulence scale length of the Dryden spectra. From the military references, a value of 2500 feet is recommended for the longitudinal turbulence scale length of the Von Kármán spectra.

## Wingspan

The wingspan is required in the calculation of the turbulence on the angular rates.

## Band-limited noise sample time (seconds)

The sample time at which the unit variance white noise signal is generated.

## Noise seeds

There are four random numbers required to generate the turbulence signals, one for each of the three velocity components and one for the roll rate. The turbulences on the pitch and yaw angular rates are based on further shaping of the outputs from the shaping filters for the vertical and lateral velocities.

## Turbulence on

Selecting the check box generates the turbulence signals.

Although no changes are needed in the number or type of block parameters, the parameters affect the subsystems of the block. The biggest effects are the subsystem changes required to simulate the selected turbulence model type.

## 4.4 Structure of COTS Block

The top level structure of the unified wind turbulence block does not change. The block is conveniently structured so that entire subsystems may be replaced to simulate the selected wind turbulence model. Figure 5 shows the highest level of the block.

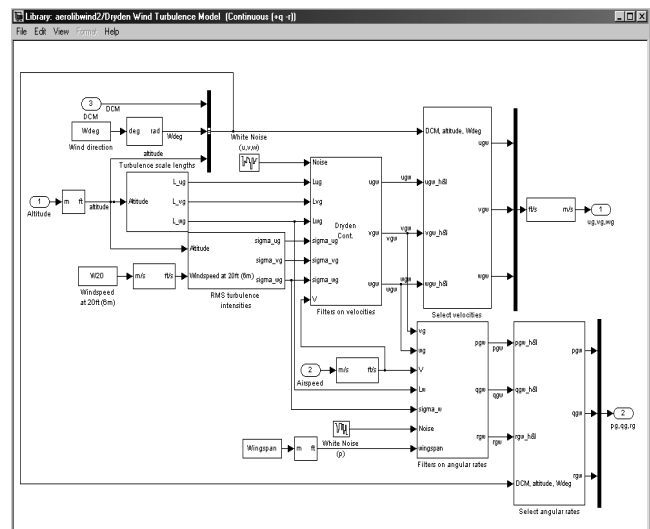


Figure 5: Top Level of Wind Turbulence Block

The replaceable subsystems are the filters on velocities subsystem and the subsystems shown in Figure 6. The

subsystems in Figure 6 are within the top level subsystem, filters on angular rates.

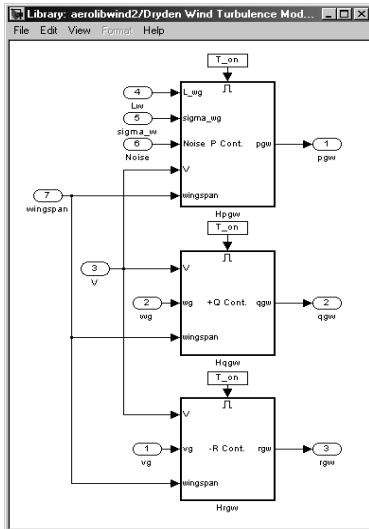


Figure 6: Filters on Angular Rates Subsystem

Figure 7 shows a library of replaceable subsystem options. The filters on velocity subsystem can be either continuous Dryden turbulence, continuous Von Kármán turbulence, or discrete Dryden turbulence. There are multiple combinations of filters on angular rates based on the sign of the turbulence angular rate spectra.

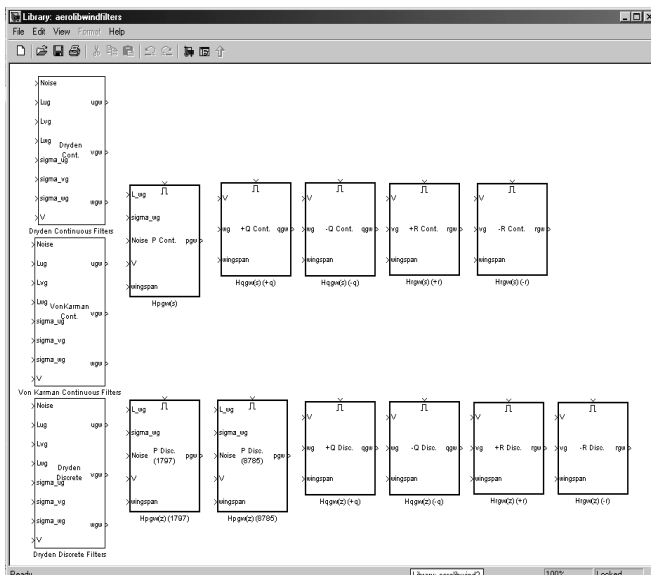


Figure 7: Library of Turbulence Filters

#### 4.5 Implementation of Interpolation Method

The unified wind turbulence block calculates both low altitude turbulence and medium/high altitude turbulence. If the altitude is above the low altitude region then the turbulence is calculated at 1000 ft or if the altitude is below the medium/high altitude region then the turbulence is calculated at 2000 ft. Within the turbulence selection subsystem, shown in Figure 8, the appropriate altitude

turbulence is selected for the block's output via a set of if-elseif-else subsystems.

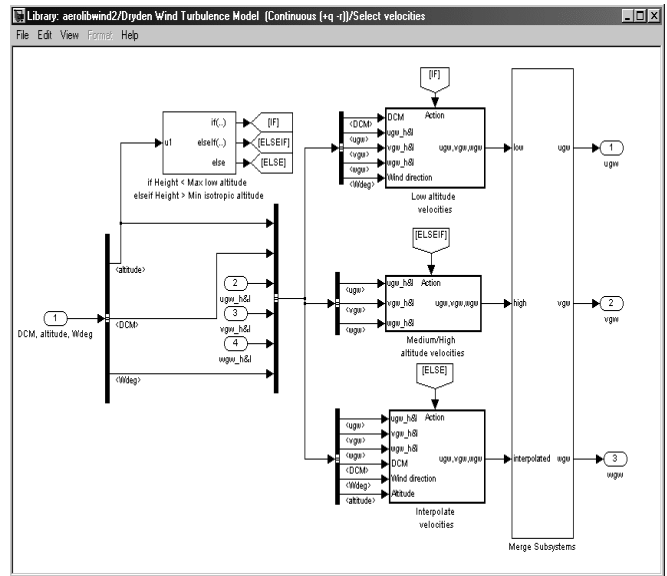


Figure 8: Turbulence Selection Subsystem

If the altitude lies within the low altitude region, the if subsystem is enabled. The if subsystem, shown in Figure 9, contains the low altitude turbulence selection and transformation to body axes.

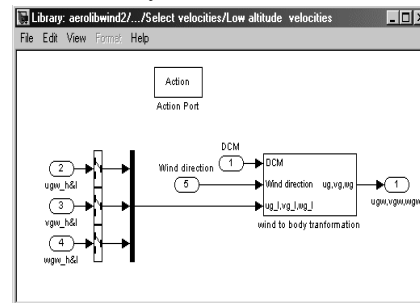


Figure 9: Low Altitude

If the altitude lies in the medium/high altitude region, the elseif subsystem is enabled. The elseif subsystem, shown in Figure 10, contains the medium/high altitude turbulence selection, which is in body axes.

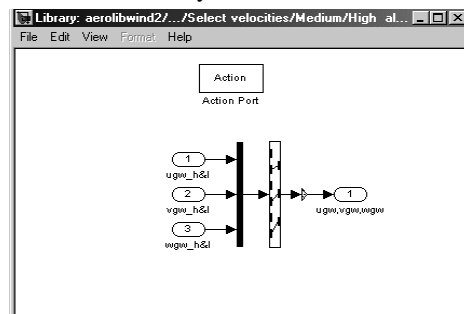


Figure 10: Medium/High Altitude

